Multiply Fractions

Fraction with Fraction

Example: Multiply and simplify $\frac{31}{35} \bullet \frac{5}{9}$

Method 1 – Multiply Then Simplify

STEP 1: Multiply straight across: numerator with numerator and denominator with denominator.

$$\frac{31}{35} \bullet \frac{5}{9} \Rightarrow \frac{155}{315}$$

Multiply across numerator and denominator.

STEP 2: Simplify, if possible. Your knowledge of *divisibility rules* is vital for simplifying this example. Since the ones place value digit of numerator and denominator is 5, then both 155 and 315 are divisible by 5. Use long division to find out how many times 5 divides into 155 (quotient is 31). Also use long division to find out how many times 5 divides into 315 (quotient is 63).

$$\frac{155}{315} \begin{matrix} \div 5 \Rightarrow \frac{31}{} \\ \div 5 \Rightarrow \frac{}{63} \end{matrix}$$

Divide by the common factor **5** to reduce.

STEP 3: Is $\frac{31}{63}$ the reduced answer or can we simplify further? Your knowledge of *prime numbers* is vital for determining if you can simplify further. Although 63 is a *composite number* and does have factors other than 1 and 63, the 31 is a prime number and does not have factors other than 1 and 31. Furthermore, 63 is not a multiple of 31. Thus, we cannot simplify further.

Answer: $\frac{31}{63}$

Method 2 – Simplify Then Multiply

STEP 1: When multiplying two fractions, the multiplication dot " • " acts as a bridge that can connect the two fractions together as one fraction. We will do that.

$$\frac{31}{35} \bullet \frac{5}{9} \Rightarrow \frac{31 \bullet 5}{35 \bullet 9}$$

Connect the two fractions as one fraction.

STEP 2: We will "break down" any *composite number* into its *prime numbers* in the numerator and denominator. The 31 and 5 in the numerator are both prime numbers so they will stay the same. However, the 35 in the denominator can be rewritten as 5 • 7. Also, the 9 in the denominator can be rewritten as 3 • 3.

$$\frac{31 \bullet 5}{35 \bullet 9} \Rightarrow \frac{31 \bullet 5}{5 \bullet 7 \bullet 3 \bullet 3}$$

"Break down" composite numbers into prime numbers.

STEP 3: We examine if we can cancel any factors that are the same between numerator and denominator. The 5 in the numerator and denominator cancel and become " 1 ". No other factors cancel.

$$\frac{31 \bullet \cancel{5}^{1}}{\cancel{5} \bullet 7 \bullet 3 \bullet 3} \quad (\text{denominator's } \cancel{5} \text{ becomes } 1)$$

Cancel the common factor **5** to simplify. Multiply remaining factors across numerator and denominator.

STEP 4: Now we multiply the remaining factors straight across in the numerator and in the denominator. Multiplying by 1 is not necessary. We can be confident that the final answer does not simplify any further. If it could, we would have seen other common factors between the numerator and denominator besides the 5 in Step 3 above.

Answer: $\frac{31}{63}$

Multiply Fractions

Whole Number with Fraction

Example: Multiply and simplify $42 \bullet \frac{6}{7}$

Note: The $42 \bullet \frac{6}{7}$ is a multiplication problem and not a *mixed number.* It is a common error to think of it as a mixed number. There is a multiplication dot " • " between the whole number (42) and the fraction $\frac{6}{7}$.

Method 1 – Multiply Then Simplify

STEP 1: We must convert the whole number into a fraction. We put a " 1 " below the 42 so that it becomes a fraction. Note that the value 42 did not change. What did change is how the 42 is presented. Now we are multiplying a fraction with fraction, and we know how to solve those problems.

$$42 \bullet \frac{6}{7} \quad \Rightarrow \quad \frac{42}{1} \bullet \frac{6}{7}$$

Put a **1** below the 42 to change it to a fraction.

STEP 2: Multiply straight across: numerator with numerator and denominator with denominator.

$$\frac{42}{1} \bullet \frac{6}{7} \quad \Rightarrow \quad \frac{252}{7}$$

Multiply across numerator and denominator.

STEP 3: Simplify, if possible. We first try *divisibility rules* to simplify but notice none of the numbers we use for divisibility apply (2, 3, 5, 9, 10). Next, we try educated guesses using other numbers to see if they simplify the answer. Since numerator (252) and denominator (7) are not both even numbers, we will skip dividing by even numbers, such as 4, 6, etc. We have already found that 3 and 5 do not work by using divisibility rules. The next odd number to check is 7. Thus, we divide both numerator and denominator by 7 and notice that 7 does divide into both. Use long division for $252 \div 7 = 36$. Finally, 36 divided by 1 is 36. Never leave a " 1 " in the denominator because the fraction simplifies.

$$\frac{252}{7} \begin{array}{c} \div 7 \\ \div 7 \end{array} \Rightarrow \frac{36}{1} \Rightarrow 36$$

Divide by the common factor 7 to reduce. Or use *long division.*

Note: In Step 3, we could have first used long division for $252 \div 7$ to see if 7 divides into 252 evenly. If we did, our quotient would be 36, which is the answer. If the numerator is very large, try dividing denominator into numerator to see if it works (remainder being 0).

Method 2 – Simplify Then Multiply

STEP 1: We must convert the whole number into a fraction. We put a " 1 " below the 42 so that it becomes a fraction. Note that the value 42 did not change. What did change is how the 42 is presented. Now we are multiplying a fraction with fraction, and we know how to solve those problems.

$$42 \bullet \frac{6}{7} \Rightarrow \frac{42}{1} \bullet \frac{6}{7}$$

Put a **1** below the 42 to change it to a fraction.

STEP 2: When multiplying two fractions, the multiplication dot " • " acts as a bridge that can connect the two fractions together as one fraction. We will do that.

$$\frac{42}{1} \bullet \frac{6}{7} \Rightarrow \frac{42 \bullet 6}{1 \bullet 7}$$

Connect the two fractions as one fraction.

STEP 3: We will "break down" any *composite number* into its *prime numbers* in the numerator and denominator. The 42 in the numerator can be rewritten as 7 • 3 • 2 (use the factor tree if needed). The 6 in the numerator can be rewritten as 3 • 2. The 7 in the denominator is prime and the 1 is neither prime nor composite so they will stay the same.

$$\frac{42 \bullet 6}{1 \bullet 7} \Rightarrow \frac{7 \bullet 3 \bullet 2 \bullet 3 \bullet 2}{1 \bullet 7}$$

"Break down" composite numbers into prime numbers.

STEP 4: Now cancel any factors that are the same between numerator and denominator. The 7 in the numerator and denominator cancel and become " 1 ". No other factors cancel.

$$\frac{\cancel{7}^{\,1} \bullet 3 \bullet 2 \bullet 3 \bullet 2}{1 \bullet \cancel{7}_{\,1}}$$

Cancel the common factor 7 to simplify. Multiply remaining factors across numerator and denominator.

STEP 5: Now we multiply the remaining factors straight across in the numerator and in the denominator. We divide 36 by 1, which gives us 36. Never leave a " 1 " in the denominator because the fraction simplifies.

Answer: $\frac{36}{1} \Rightarrow 36$