Prime Factorization

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***Definitions*:**

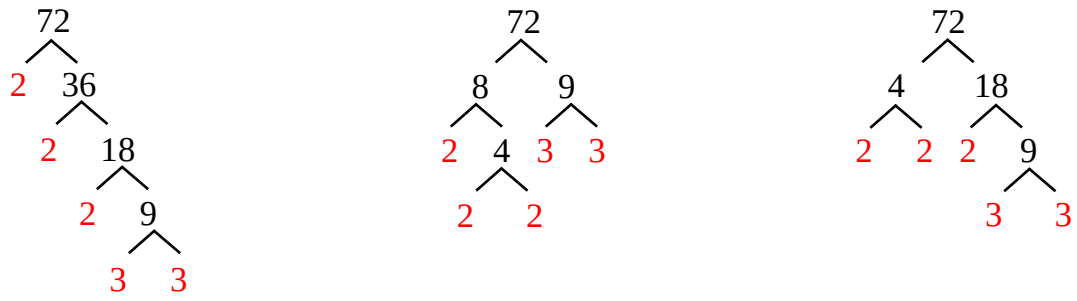
- **Factors** are numbers that when multiplied together, result in another number. For example, 2 and 5 are factors of 10, because $2 \cdot 5 = 10$.
- A **prime number** is a natural number (1, 2, 3, etc.) having two *different* factors, itself and 1.
  - The number '1' is not a prime number (since it does not have two *different* factors).
  - If a natural number (other than 1) is not a prime number, then it is called a composite number. A **composite number** can be broken down into prime numbers.
    - For example, 4 (can be broken down into $2 \cdot 2$) and 9 (can be broken down into $3 \cdot 3$) are composite numbers.
  - The prime numbers up to 101 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101…
- **Prime factorization** refers to factoring a composite number into a product of its prime numbers. For example, the prime factorization of 12 is $2 \cdot 2 \cdot 3$.

Finding Prime Factorizations

Here are two methods for finding prime factorizations: The "Factor Tree Method" and the "Division Up Method".

"**Factor Tree Method**" – continuously factor using prime factors and composite numbers until all there is left are prime number factors.

Example: Find the prime factorization of 72.



Notice that it does not matter which numbers you factor along the way (which 'path' you take). When finished, the same factors will be obtained. The prime factorization of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

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"**Division Up Method**" – divide by prime numbers beginning with the lowest prime number, 2, using the "division up" technique shown below.

Example: Find the prime factorization of 72.

Step 1: Divide 72 by the first prime, 2. The 2 divides evenly (no remainder) into 72 exactly 36 times.

$$\begin{array}{r|l} & 36 \\ \hline 2 & 72 \end{array}$$

Step 2: Divide 36 by the first prime again, 2. The 2 divides evenly into 36 exactly 18 times.

$$\begin{array}{r|l} & 18 \\ \hline 2 & 36 \\ \hline 2 & 72 \end{array}$$

Step 3: Divide 18 by the first prime again, 2. The 2 divides evenly into 18 exactly 9 times.

$$\begin{array}{r|l} & 9 \\ \hline 2 & 18 \\ \hline 2 & 36 \\ \hline 2 & 72 \end{array}$$

Step 4: Dividing 9 by the first prime 2 will result in a remainder. Therefore divide 9 by the next higher prime number, 3. The 3 divides evenly into 9 exactly 3 times. The quotient is a prime number – stop dividing. When the quotient is a prime number, prime factorization is complete.

$$\begin{array}{r|l} & 3 \\ \hline 3 & 9 \\ \hline 2 & 18 \\ \hline 2 & 36 \\ \hline 2 & 72 \end{array}$$

The prime factorization of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$