LCM and LCD

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Lecture Notes

Definitions

- A **multiple** is the *product* of two factors. Ex: $\mathbf{1} \cdot 7 = \mathbf{7}$, $\mathbf{2} \cdot 7 = \mathbf{14}$, $\mathbf{3} \cdot 7 = \mathbf{21}$, etc.
- The **Least Common Multiple (LCM)** is the smallest multiple that two factors divide into.
  - The LCM is the smallest multiple that is divisible by two factors.
  - The LCM is a *whole number*.
  - The LCM **is not** the smallest number that divides into two numbers – a common mistake.
- The **Lowest Common Denominator (LCD)** has the same definition as the LCM, except the LCD refers to the *denominator* of a fraction.
  - Finding the LCD of two fractions allows us to add or subtract the fractions although they originally had two different denominators.

Two Methods for Finding the LCM (or LCD)

Method 1: One List of Multiples

- **Step 1:** Does the smaller number divide evenly (with 0 remainder) into the bigger number?
  - If it does, the *bigger number is the LCM* (or LCD).
- **Step 2:** Find the **second multiple** of the **bigger number**.
  - Does the smaller number divide evenly into the *second multiple* of the bigger number?
  - If it does, the second multiple of the bigger number is the LCM.
- **Step 3:** Find the **third multiple** of the **bigger number**.
  - Does the smaller number divide evenly into the *third multiple* of the bigger number?
  - If it does, the third multiple of the bigger number is the LCM.
- *Notes:*
  - The LCM is either the *bigger* of the two given numbers, or *bigger*.
  - If the LCM has not been found after the third multiple of the bigger number, switch over to the *factor tree method* instead.

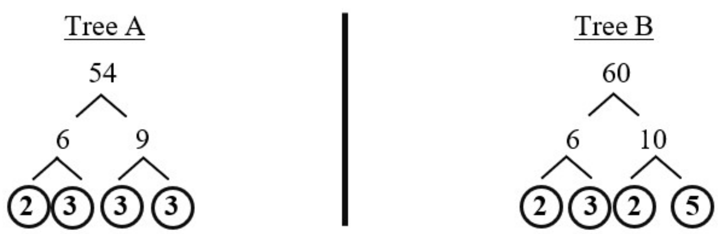
Example with One List of Multiples: Find the LCM of 6 and 8.

- *Step 1:* Does 6 divide into 8? No.
- *Step 2:* Find the **second multiple** of 8: $\mathbf{2} \cdot 8 = \mathbf{16}$
  - Does 6 divide into 16? No.
- *Step 3:* Find the **third multiple** of 8: $\mathbf{3} \cdot 8 = \mathbf{24}$
  - Does 6 divide into 24? Yes. The LCM is **24**.
- *Note:* The smallest number possible that *both* 6 and 8 divide into is **24**.

Method 2: Factor Tree

- **Step 1:** Find the prime factorization of the two numbers. See example below.
- **Step 2:** Compare the *number of occurrences* of each prime number between Tree A and Tree B.
- **Step 3:** The side with the **most occurrences** of the prime number being compared will become part of the LCM (or LCD).
    - The **most** occurrences of the number 2 are two. There are **more** 2's in Tree B so "bring down" those two 2's to start forming the LCM. So far we have: LCM = **2 • 2**
    - Leave the 2 in Tree A. Think of it as, "The winner comes down and the loser stays back."
- **Step 4:** Continue the process above by comparing the *number of occurrences* of each prime number between Tree A and Tree B.
    - After comparing all the prime numbers, we have LCM = 2 • 2 • 3 • 3 • 3 • 5 = **540**
    - Make sure to multiply the prime numbers from factored form to get the LCM: 540.
- *Notes:*
    - When comparing prime numbers between both sides and there is a **tie** in the number of occurrences, "bring down" the prime number(s) from **either side**, but **not from both sides**.
    - Draw a vertical "divider" line to separate the two different factor trees, as shown below.

Example with Factor Tree: Find the LCM of 54 and 60.



Notes

- I recommend that you **always start with the One List of Multiples** method regardless. If after trying a few multiples you did not find the LCM, switch over to the *factor tree method*.
    - The *One List of Multiples* method works best when it is easy to visually notice that the given numbers will match within the first few multiples.
- Conversely, start with the **Factor Tree** method if the two numbers are "awkward" (9 and 11) or big (25 and 35).
    - Using the *One List of Multiples* method for these types of numbers can become very time-consuming and you might be working with difficult numbers.
- The process of finding the LCM and LCD is identical.
- When denominators of two fractions are different, it is because their *unit fractions* are different.
    - When we try to add (or subtract) fractions with different denominators, we can't.
    - Think of it as if the two fractions are speaking different languages.
        - Ex: One fraction speaks in "tenths" while the other speaks in "fifteenths".
    - We must find a common language for them to speak.
    - That common language is found when they have the **same unit fraction**.
    - This means having the same number as their denominator: the **LCD**.

Find the prime factorization of the numbers. Then find the LCM.

50, 56

The prime factorization of 50 is $2 \cdot 5 \cdot 5$.

The prime factorization of 56 is $2 \cdot 2 \cdot 2 \cdot 7$.

The LCM is 1400.

- The number 50 and 56 are big. Do not use the *One List of Multiples* method.
- The *Factor Tree* method is the obvious choice.

Find the LCM of this set of numbers. Do so mentally if possible. 7 and 21	The LCM is 21.

- The numbers 7 and 21 are small numbers. Also, they are in the multiplication facts table.
- Definitely use the *One List of Multiples* method.
- Does 7 divide into 21?
  - Yes. The bigger number is the LCM.
  - Thus, LCM = **21**.

Find the least common multiple of the set of numbers.

2,5

The least common multiple of 2 and 5 is 10.

- When the two given numbers are **different** prime numbers, simply *multiply* the two numbers to get the LCM: $2 \cdot 5 = \mathbf{10}$
- Knowing this "shortcut technique" can save you time.
- You can always use the *One List of Multiples* or the *Factor Tree* method to find the LCM.

Find the LCM of this set of numbers. Do so mentally if possible. 3 and 6	The LCM is 6.

- Always start with the *One List of Multiples* method.
- Does 3 divide into 6?
  - Yes. The bigger number is the LCM.
  - Thus, LCM = **6**.

Find the least common multiple of the set of numbers.

4,7

The least common multiple of 4 and 7 is 28.

- The prime factorization of 4 is $\mathbf{2 \cdot 2}$
- The prime factorization of 7 is just **7** itself.
- Since we have **different** prime factors **between** the two given numbers, we'll use the "shortcut technique" discussed above.
    - Knowing this "shortcut technique" can save you time.
- Multiply the two numbers to get the LCM: $4 \cdot 7 = \mathbf{28}$

Use multiples of the larger number to find the least common multiple in the set of numbers.

6 and 34

LCM = 102 (Type a whole number.)

- Always start with the *One List of Multiples* method.
- Does 6 divide into 34? No.
- Find the **second multiple** of 34: $\mathbf{2} \cdot 34 = \mathbf{68}$
- Does 6 divide into 68? No.
- Find the **third multiple** of 34: $\mathbf{3} \cdot 34 = \mathbf{102}$
- Does 6 divide into 102? Yes. The LCM is **102**.
- In this problem, multiples of 34 were quickly getting big. If after the third multiple we still had not found the LCM, we would have switched over to the *Factor Tree* method.

Use multiples of the larger number to find the LCM of this set of numbers. 15 and 25	The LCM of 15 and 25 is 75.

- Although we would have found the LCM on the third multiple of 25, constantly dividing by 15 is not easy to do, right?
- For this problem, it would be better to start with the *Factor Tree* method.

Find the LCM of this set of numbers. Do so mentally if possible. 6 and 30	The LCM of 6 and 30 is 30.

- Which method would you use, the *One List of Multiples* or the *Factor Tree*, and why?

Find the LCD for the following pair of fractions.

$\frac{1}{8}$ and $\frac{4}{7}$

The least common denominator is 56 .

- The process of finding the LCM and LCD is identical.
    - We are finding the LCM of the two denominators: 8 and 7.
    - In this problem, it is called LCD because we are dealing with denominators of fractions.
- To find the LCD, which of the two methods will you use?
    - *One List of Multiples* method?
    - *Factor Tree* method?
    - Is it possible to use the “shortcut technique” for this problem?
- Which of the three procedures would be the most time-consuming for the given denominators?
    - *One List of Multiples* method?
    - *Factor Tree* method?
    - The “shortcut technique”?

Write the fractions as equivalent fractions with the LCD.

$\frac{9}{10}$ and $\frac{5}{8}$

The equivalent fractions with the LCD are $\frac{36}{40}, \frac{25}{40}$.

(Use a comma to separate answers.)

- For this problem, we will use the same procedure as in the previous one.
- However, we are asked to find **equivalent fractions** based on the LCD.
- Which method will we use to find the LCD between 10 and 8, and why?
- Is it possible to use the "shortcut technique" here?
- First we find the LCD, and it is **2 • 2 • 2 • 5** = **40**.
- Then, we will a procedure similar to the one from the *Equivalent Fractions* section. Do you remember this type of problem below? We will find equivalent fractions in a similar way.

  Find the missing numerator so that the fractions will be equal.

  $\frac{5}{7} = \frac{?}{42}$

  $\frac{5}{7} = \frac{30}{42}$

- Start by placing a multiplication **dot** '•' in front of the *left* denominator: $\frac{9}{\bullet\ 10}$
- Ask yourself, "What number times 10 equals the LCD, **40**?"
  - That factor is **4**.
  - Therefore, multiply *both* the numerator *and* denominator of left fraction by **4**: $\frac{4 \bullet 9}{4 \bullet 10}$
  - The equivalent left fraction becomes $\frac{\mathbf{36}}{\mathbf{40}}$
- Then place a multiplication **dot** '•' in front of the *right* denominator: $\frac{5}{\bullet\ 8}$
- Ask yourself, "What number times 8 equals the LCD, **40**?"
  - That factor is **5**.
  - Therefore, multiply *both* the numerator *and* denominator of left fraction by **5**: $\frac{5 \bullet 5}{5 \bullet 8}$
  - The equivalent left fraction becomes $\frac{\mathbf{25}}{\mathbf{40}}$
- The answer, with a comma between the two fractons, is: $\frac{\mathbf{36}}{\mathbf{40}}, \frac{\mathbf{25}}{\mathbf{40}}$
- <u>*Notes*</u>:
  - After placing the multiplication dot '•' in front of a denominator and asking yourself, "What number times the denominator equals the LCD," you can use the *factored form* of the prime factorization to help you: **2 • 2 • 2 • 5**
  - As mentioned previously, always read the additional instructions in blue to see the format of the answer that is expected. In this problem, a comma is needed to separate the fractions.