FRACTIONS WORKBOOK

Convert Mixed Number to Improper Fraction

Example: $15\frac{3}{8}$ ⇨ $\frac{120 + 3}{8}$ ⇨ $\frac{123}{8}$

STEP 1 – Multiply: Multiply denominator (8) and whole number (15) to get a temporary result of 120.

STEP 2 – Add: Add temporary result of 120 to numerator (3). That answer (123) becomes the new numerator. The denominator (8) stays the same.

Note: The $15\frac{3}{8}$ is a *mixed number,* not a multiplication problem. It is a common error to think of it as multiplication. There is no multiplication symbol indicating multiplication between the whole number (15) and the fraction $\frac{3}{8}$.

Convert Improper Fraction to Mixed Number

Example: $\frac{123}{8}$ ⇨

$$\begin{array}{r} 15 \\ 8\overline{)123} \\ -\ 8\downarrow \\ \hline 43 \\ -\ 40 \\ \hline 3 \end{array}$$

⇨ $15\frac{3}{8}$

Use *long division*.

STEP 1 – Divide: Do long division.

a. The **Q**uotient (15) is the whole number part of the answer at the top.
b. The **R**emainder (3) is how much is left over after the last subtraction step.
c. The **D**ivisor (8) is the number we are dividing by.

STEP 2 – Create Mixed Number: Use the result from long division to create the mixed number. The format of a mixed number is shown below:

$$\textit{Quotient}\ (15)\ \frac{\textit{Remainder}\ (3)}{\textit{Divisor}\ (8)}$$

Remember format as $Q\frac{R}{D}$

Note: Recall that a fraction means division. Divide the denominator *into* the numerator. However, first change the format into *long division.*

Find All the Factors of a Number

Example: Find *all* the factors of 250.

STEP 1: Start with the number 1 as a factor.

$1 \bullet 250 = 250$ — Two factors listed.

STEP 2: Using *divisibility rules,* ask yourself if 250 is divisible by 2. Yes, it is because 250 is an even number. Use long division to find out how many times 2 divides into 250. The answer to long division (quotient) will be the other factor (125). So far, we have four factors: $1, 2, 125, 250$. But there could be more, so we continue.

$1 \bullet 250 = 250$

$2 \bullet 125 = 250$ — Four factors listed.

STEP 3: Using divisibility rules, ask yourself if 250 is divisible by 3. No, it is not because the sum of the digits of 250 is not divisible by 3. Skip the factor 3.

STEP 4: Although there is a divisibility rule for the number 4, we do not cover it in this course. Use long division to see if 4 divides *evenly* into 250. If it does, then the quotient is the other factor. However, 4 does not divide evenly into 250. Skip the factor 4.

STEP 5: Using divisibility rules, ask yourself if 250 is divisible by 5. Yes, it is because the ones place value digit of 250 is 0 (divisible if ones is either 0 or 5). Use long division to find out how many times 5 divides into 250. The quotient for long division is 50. Therefore, 50 will be the other factor. We are up to six factors: $1, 2, 5, 50, 125, 250$. But there could be *two more* factors, so we continue.

$1 \bullet 250 = 250$

$2 \bullet 125 = 250$ — Six factors listed.

$5 \bullet \ \ 50 = 250$

STEP 6: While we can proceed sequentially checking the numbers 6, 7, 8, and 9, notice that the number 250 ends in 0. And divisibility rules states that if the ones place value digit is 0, then 10 will divide into that number evenly. Use long division to find out how many times 10 divides into 250. The quotient is 25. Therefore, 25 will be the other factor. We have reached *eight factors,* so we are done: $1, 2, 5, 10, 25, 50, 125, 250$.

$1 \bullet 250 = 250$

$2 \bullet 125 = 250$

$5 \bullet \ \ 50 = 250$ — Eight factors listed. Done.

$10 \bullet \ \ 25 = 250$

Answer: $1, 2, 5, 10, 25, 50, 125, 250$

See Notes on next page.

Notes for Finding All the Factors of a Number:

- Notice that the directions say to find *all* the factors of 250. That includes prime numbers, composite numbers, and the number 1 (neither prime nor composite).
- Do not use *prime factorization* (factor tree) for this type of problem. The factor tree will be shown next for a different type of problem.
- Instead, create a list using a systematic process to find all the factors. Start with the factor 1 and continue up in sequential numerical order.
- *Divisibility rules* played a pivotal role in helping us to find all the factors.
- It is helpful to know that for this course there will be 8 total factors, or fewer. Thus, if you list 8 factors, do not look for more. However, there could be fewer than 8 factors.
- If the number is large, such as 250 or 375, it can be useful to think of it as money. For example, 250 would be equivalent to $2.50 and then you can use your knowledge of money coins to help you find some factors. If you had enough of a particular coin, could you get $2.50? You could with these coins: 1¢, 5¢, 10¢, and 25¢. Similarly, 1, 5, 10, and 25 are factors of 250.

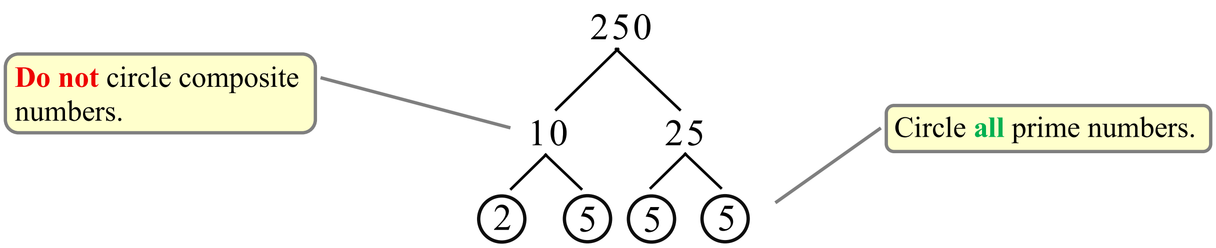
Find Prime Factorization

Example: Find the *prime factorization* of 250.

STEP 1: Begin a *factor tree* by writing 250 at the top and two branches below it.

STEP 2: Ask yourself, “What number times what number is 250?” Use *divisibility rules* to find the two factors. We have choices because 250 is divisible by 2, 5, and 10. We choose 10 to quickly reduce 250. To find the other factor, divide 10 into 250 to get 25.

STEP 3: Now we have the *composite numbers* 10 and 25. Continue down the factor tree by finding the factors of 10 and 25. The factors of 10 are 2 and 5, which are both *prime numbers,* so we circle them. The factors of 25 are 5 and 5, which are both prime numbers and we circle them.

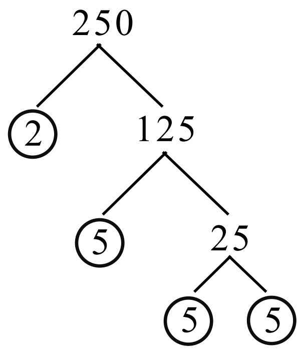


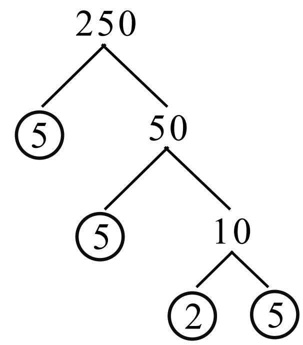
Use the circled prime numbers in the factor tree to list prime factorization: 2 • 5 • 5 • 5

Notes:

- Notice that the instructions say to find the *prime factorization* of 250, which means we are only concerned with the *prime numbers* of 250. We will exclude composite numbers and the number 1, which is neither prime nor composite.
- Knowledge of prime numbers is necessary to solve this type of problem, which is why you must memorize prime numbers 2 through 47.
- Circle the prime numbers as you go down the factor tree. Do not circle the composite numbers.
- We continue down the factor tree until the end of each branch has a circled prime number.
- When the end of every branch has a circled prime number, we are done using the factor tree.
- The answer includes the multiplication dot “ • ” between each prime number because the format of *prime factorization* must be in *factored form.*
- Although not required, list the prime numbers from smallest to largest.
- Notice that the answer to this problem is in *prime factorization* format as compared to the previous problem of finding *all* the factors of a number which is in a list format separated by commas. The answers (numbers and formats) are completely different. It is a common mistake to confuse these two type of problems.

- For the factor tree, there is no right or wrong way on which factors you choose at each branch. For example, if you started the initial branch with the alternate number combinations shown below, you would still arrive at the same prime factorization.





The prime factorization is the same: 2 • 5 • 5 • 5

Simplify a Fraction

Example: Simplify the fraction $\frac{24}{30}$.

To *simplify* a fraction means to *reduce* both the numerator and the denominator by a common factor. We are making the fraction smaller, if it is possible.

Method 1 – Divide by a Common Factor

STEP 1: Find a number that divides into *both* 24 and 30. Ideally you want to find the biggest number for the maximum reduction. Although the 24 and 30 are in the multiplication facts table and you should know that 6 divides into them, pretend that you do not know for this example. If you cannot think of the biggest number, then use *divisibility rules* to help you. The 24 and 30 are even numbers so 2 divides into them. Thus, we divide the numerator and denominator by 2 to reduce the fraction.

$$\frac{24 \div 2}{30 \div 2} \Rightarrow \frac{12}{15}$$

Divide by the common factor **2** to reduce.

STEP 2: Now we have $\frac{12}{15}$. Ask yourself, "Is this fraction fully reduced, or can it be further simplified?" Repeat the step above to find a number that divides into *both* 12 and 15. You can use your knowledge of the multiplication facts or divisibility rules to find that number. The number that divides into 12 and 15 is 3. We divide the numerator and denominator by 3 to reduce further.

$$\frac{24 \div 2}{30 \div 2} \Rightarrow \frac{12 \div 3}{15 \div 3} \Rightarrow \frac{4}{5}$$

Divide by the common factor **3** to reduce.

STEP 3: Now we have $\frac{4}{5}$. Ask yourself, "Is this fraction fully reduced, or can it be further simplified?" Repeat the step above to find a number that divides into *both* 4 and 5. The only number that divides into 4 and 5 is 1, thus we are done simplifying. If we divided by 1, we would get the same fraction back, $\frac{4}{5}$.

Tip: To reduce quickly, think of the biggest number that divides into both the numerator and denominator. Here, the biggest number that divides into $\frac{24}{30}$ is 6. Thus,

$$\frac{24 \div 6}{30 \div 6} \Rightarrow \frac{4}{5}$$

Divide by the biggest common factor **6** to reduce.

Method 2 – Find Prime Factorization Then Cancel

STEP 1: Find prime factorization of both 24 and 30. Use the factor tree as needed.

$$\frac{24}{30} \Rightarrow \frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 5}$$

List the prime numbers of **24** and **30**.

STEP 2: Find any common factors between the numerator and denominator. If there are common factors, then you can "cancel" them. *Cancelling* means the factors become 1. Why? Because a fraction means division. And any number divided by itself is 1. Thus, we can cancel the 2 from both the numerator and denominator. We can also cancel the 3 from both the numerator and denominator. Slash out the 2 and 3 and put a " 1 " in their place.

$$\frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 5} \Rightarrow \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 5}$$

Cancel common factors, **2** and **3**, from numerator and denominator.

STEP 3: Ensure there are no more common factors between numerator and denominator. Then multiply the remaining factors straight across in the numerator and in the denominator. Do not bother multiplying the cancelled factors " 1 " because 1 times anything results in that other number.

$$\frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 5} \Rightarrow \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 5} \Rightarrow \frac{4}{5}$$

Multiply remaining factors across numerator and denominator.

Notes:

- Factors between numerator and denominator do not need to be directly over each other to cancel them. Since the factors are connected by the multiplication dot " • " they can be located anywhere along the numerator or the denominator. This is due to the *Commutative Property of Multiplication.*
- You cannot cancel the same factors that appear together in the numerator or the same factors that appear together in the denominator.
- You can only cancel "up and down" or "diagonally" *across* the fraction bar. You cannot cancel "side to side" on the same side of the fraction bar.

Multiply Fractions

Fraction with Fraction

Example: Multiply and simplify $\frac{31}{35} \bullet \frac{5}{9}$

Method 1 – Multiply Then Simplify

STEP 1: Multiply straight across: numerator with numerator and denominator with denominator.

$$\frac{31}{35} \bullet \frac{5}{9} \Rightarrow \frac{155}{315}$$

Multiply across numerator and denominator.

STEP 2: Simplify, if possible. Your knowledge of *divisibility rules* is vital for simplifying this example. Since the ones place value digit of numerator and denominator is 5, then both 155 and 315 are divisible by 5. Use long division to find out how many times 5 divides into 155 (quotient is 31). Also use long division to find out how many times 5 divides into 315 (quotient is 63).

$$\frac{155 \div 5}{315 \div 5} \Rightarrow \frac{31}{63}$$

Divide by the common factor **5** to reduce.

STEP 3: Is $\frac{31}{63}$ the reduced answer or can we simplify further? Your knowledge of *prime numbers* is vital for determining if you can simplify further. Although 63 is a *composite number* and does have factors other than 1 and 63, the 31 is a prime number and does not have factors other than 1 and 31. Furthermore, 63 is not a multiple of 31. Thus, we cannot simplify further.

Answer: $\frac{31}{63}$

Method 2 – Simplify Then Multiply

STEP 1: When multiplying two fractions, the multiplication dot " • " acts as a bridge that can connect the two fractions together as one fraction. We will do that.

$$\frac{31}{35} \bullet \frac{5}{9} \Rightarrow \frac{31 \bullet 5}{35 \bullet 9}$$

Connect the two fractions as one fraction.

STEP 2: We will "break down" any *composite number* into its *prime numbers* in the numerator and denominator. The 31 and 5 in the numerator are both prime numbers so they will stay the same. However, the 35 in the denominator can be rewritten as 5 • 7. Also, the 9 in the denominator can be rewritten as 3 • 3.

$$\frac{31 \bullet 5}{35 \bullet 9} \Rightarrow \frac{31 \bullet 5}{5 \bullet 7 \bullet 3 \bullet 3}$$

"Break down" composite numbers into prime numbers.

STEP 3: We examine if we can cancel any factors that are the same between numerator and denominator. The 5 in the numerator and denominator cancel and become " 1 ". No other factors cancel.

$$\frac{31 \bullet \cancel{5}^{\,1}}{\cancel{5}_{\,1} \bullet 7 \bullet 3 \bullet 3}$$

Cancel the common factor **5** to simplify. Multiply remaining factors across numerator and denominator.

STEP 4: Now we multiply the remaining factors straight across in the numerator and in the denominator. Multiplying by 1 is not necessary. We can be confident that the final answer does not simplify any further. If it could, we would have seen other common factors between the numerator and denominator besides the 5 in Step 3 above.

Answer: $\frac{31}{63}$

Multiply Fractions

Whole Number with Fraction

Example: Multiply and simplify $42 \bullet \frac{6}{7}$

Note: The $42 \bullet \frac{6}{7}$ is a multiplication problem and not a *mixed number.* It is a common error to think of it as a mixed number. There is a multiplication dot " • " between the whole number (42) and the fraction $\frac{6}{7}$.

Method 1 – Multiply Then Simplify

STEP 1: We must convert the whole number into a fraction. We put a " 1 " below the 42 so that it becomes a fraction. Note that the value 42 did not change. What did change is how the 42 is presented. Now we are multiplying a fraction with fraction, and we know how to solve those problems.

$$42 \bullet \frac{6}{7} \Rightarrow \frac{42}{1} \bullet \frac{6}{7}$$

Put a **1** below the 42 to change it to a fraction.

STEP 2: Multiply straight across: numerator with numerator and denominator with denominator.

$$\frac{42}{1} \bullet \frac{6}{7} \Rightarrow \frac{252}{7}$$

Multiply across numerator and denominator.

STEP 3: Simplify, if possible. We first try *divisibility rules* to simplify but notice none of the numbers we use for divisibility apply (2, 3, 5, 9, 10). Next, we try educated guesses using other numbers to see if they simplify the answer. Since numerator (252) and denominator (7) are not both even numbers, we will skip dividing by even numbers, such as 4, 6, etc. We have already found that 3 and 5 do not work by using divisibility rules. The next odd number to check is 7. Thus, we divide both numerator and denominator by 7 and notice that 7 does divide into both. Use long division for $252 \div 7 = 36$. Finally, 36 divided by 1 is 36. Never leave a " 1 " in the denominator because the fraction simplifies.

$$\frac{252 \div 7}{7 \div 7} \Rightarrow \frac{36}{1} \Rightarrow 36$$

Divide by the common factor 7 to reduce. Or use *long division.*

Note: In Step 3, we could have first used long division for $252 \div 7$ to see if 7 divides into 252 evenly. If we did, our quotient would be 36, which is the answer. If the numerator is very large, try dividing denominator into numerator to see if it works (remainder being 0).

Method 2 – Simplify Then Multiply

STEP 1: We must convert the whole number into a fraction. We put a " 1 " below the 42 so that it becomes a fraction. Note that the value 42 did not change. What did change is how the 42 is presented. Now we are multiplying a fraction with fraction, and we know how to solve those problems.

$$42 \cdot \frac{6}{7} \Rightarrow \frac{42}{1} \cdot \frac{6}{7}$$

> Put a **1** below the 42 to change it to a fraction.

STEP 2: When multiplying two fractions, the multiplication dot " • " acts as a bridge that can connect the two fractions together as one fraction. We will do that.

$$\frac{42}{1} \cdot \frac{6}{7} \Rightarrow \frac{42 \cdot 6}{1 \cdot 7}$$

> Connect the two fractions as one fraction.

STEP 3: We will "break down" any *composite number* into its *prime numbers* in the numerator and denominator. The 42 in the numerator can be rewritten as 7 • 3 • 2 (use the factor tree if needed). The 6 in the numerator can be rewritten as 3 • 2. The 7 in the denominator is prime and the 1 is neither prime nor composite so they will stay the same.

$$\frac{42 \cdot 6}{1 \cdot 7} \Rightarrow \frac{7 \cdot 3 \cdot 2 \cdot 3 \cdot 2}{1 \cdot 7}$$

> "Break down" composite numbers into prime numbers.

STEP 4: Now cancel any factors that are the same between numerator and denominator. The 7 in the numerator and denominator cancel and become " 1 ". No other factors cancel.

$$\frac{\overset{1}{\cancel{7}} \cdot 3 \cdot 2 \cdot 3 \cdot 2}{1 \cdot \underset{1}{\cancel{7}}}$$

> Cancel the common factor **7** to simplify. Multiply remaining factors across numerator and denominator.

STEP 5: Now we multiply the remaining factors straight across in the numerator and in the denominator. We divide 36 by 1, which gives us 36. Never leave a " 1 " in the denominator because the fraction simplifies.

Answer: $\frac{36}{1} \Rightarrow 36$

Divide Fractions

Fraction with Fraction

Example: Divide and simplify $\frac{8}{3} \div \frac{32}{7}$

STEP 1: The first step is to convert the division problem into a multiplication problem. We use the **K**eep **C**hange **F**lip technique to do that.

$$\frac{8}{3} \div \frac{32}{7} \Rightarrow \frac{8}{3} \bullet \frac{7}{32}$$

Keep left fraction the same.

Change division to multiplication.

Flip the right fraction.

We changed the problem from division to multiplication.

Keep
Change
Flip

STEP 2: Now we treat the problem as multiplication. As noted in the multiplication problem earlier, we have a choice in how we proceed. We can use either "Method 1 – Multiply Then Simplify" or "Method 2 – Simplify Then Multiply." Let's simplify first and then multiply. We connect the two fractions together. Then we find prime factorization for all composite numbers in the numerator and the denominator.

$$\frac{8}{3} \bullet \frac{7}{32} \Rightarrow \frac{2 \bullet 2 \bullet 2 \bullet 7}{3 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2}$$

"Break down" composite numbers into prime numbers.

STEP 3: Next, we cancel common factors (2) between the numerator and denominator. Recall that the factors do not need to be directly over each other. Multiply the remaining factors straight across in the numerator and denominator for the simplified answer.

$$\frac{\overset{1}{\cancel{2}} \bullet \overset{1}{\cancel{2}} \bullet \overset{1}{\cancel{2}} \bullet 7}{3 \bullet 2 \bullet 2 \bullet \underset{1}{\cancel{2}} \bullet \underset{1}{\cancel{2}} \bullet \underset{1}{\cancel{2}}} \Rightarrow \text{Answer: } \frac{7}{12}$$

Cancel the common factor **2** to simplify. Multiply remaining factors across numerator and denominator.

Divide Fractions

Whole Number with Fraction

Example: Divide and simplify $21 \div \frac{3}{4}$

STEP 1: We convert the whole number into a fraction by putting a " 1 " below the 21. Now we are dividing a fraction with fraction, and we know how to solve those problems. Next, we perform **K**eep **C**hange **F**lip to convert the division problem into a multiplication problem.

$$21 \div \frac{3}{4} \Rightarrow \frac{21}{1} \div \frac{3}{4} \Rightarrow \frac{21}{1} \bullet \frac{4}{3}$$

Put a **1** below the 21 to change it to a fraction. Then do **K**eep **C**hange **F**lip.

STEP 2: We have a choice to use either "Method 1 – Multiply Then Simplify" or "Method 2 – Simplify Then Multiply." This time let's multiply then simplify (for practice). Multiply straight across, numerator with numerator and denominator with denominator.

$$\frac{21}{1} \bullet \frac{4}{3} \Rightarrow \frac{84}{3}$$

Multiply across numerator and denominator.

STEP 3: Simplify, if possible. Use your knowledge of *divisibility rules* to find a common factor that divides evenly into both numerator and denominator. We find that 3 divides into both 84 and 3. We divide numerator and denominator by 3 to get $\frac{28}{1}$. However, never leave a " 1 " in the denominator as the final answer because we can divide 28 by 1, which results in 28.

$$\frac{84 \div 3}{3 \div 3} \Rightarrow \frac{28}{1} \Rightarrow \text{Answer: } 28$$

Divide by the common factor **3** to reduce. Or use *long division.*

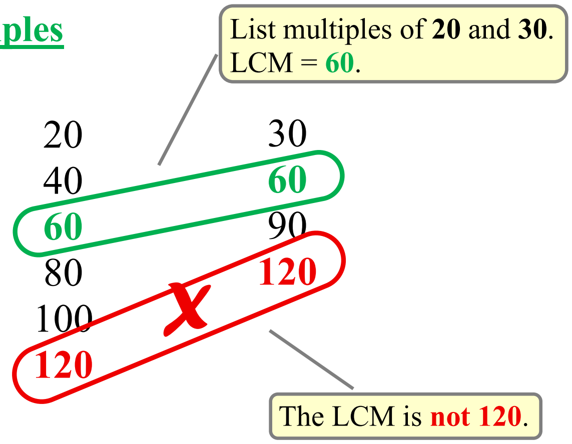
Note: Another way to reduce is to first try *long division* with $\frac{84}{3}$. Divide the denominator (3) *into* the numerator (84) to see if the remainder is 0. It is 0 and therefore the quotient (28) is the answer. Since the numerator is very large, we can try dividing the denominator *into* the numerator to see if it works (remainder being 0). And it does.

Find Least Common Multiple (LCM)

Example: Find the LCM of this set of numbers: 20 and 30.

The **L**east **C**ommon **M**ultiple (**LCM**) of two numbers is the smallest possible value that both numbers can divide into. Thus, the LCM is either the biggest of the two numbers, or bigger. A common mistake is to think of the LCM as the smallest value that divides into the two numbers. But it is the other way around.

Method 1 – List of Multiples



STEP 1: Write multiples of 20 and 30.

Note: A *multiple* means to repeatedly add a number. But here is another way to think of multiples:

1 • 20 = 20
2 • 20 = 40
3 • 20 = **60**
Etc.

1 • 30 = 30
2 • 30 = **60**
Etc.

STEP 2: Look for the smallest number that is common to both multiples. That number is 60 and represents the *least common multiple* of 20 and 30. Notice that 120 also is a common multiple of 20 and 30. However, it is not the least common multiple. Start your list of multiples with the smaller (20) number to see if you get a quick match with the bigger number (30). If there is no quick match, then do multiples – back and forth between 20 and 30 – until you get a match. This alternating listing of multiples, although optional, helps to avoid creating unnecessary multiples.

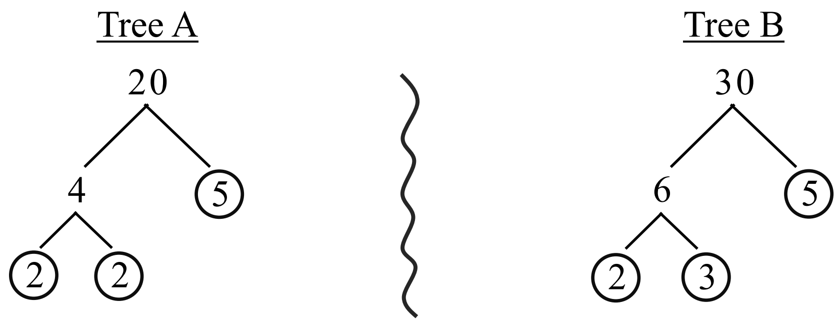
LCM: 60

Notes:

- Another method called *Shares & Leftovers* is explained next. Use the following guidance for determining which method to use.
  - When finding the LCM of relatively easy multiples, such as 20 and 30, it is easier to use the *List of Multiples* method.
  - When finding the LCM of difficult multiples, such as 27 and 45, it is easier to use the *Shares & Leftovers* method.
- When explaining the *Shares & Leftovers* method, we will use the LCM of 20 and 30 again to illustrate the difference between the two methods.

<u>**Method 2 – Shares & Leftovers**</u>

<u>STEP 1</u>: Complete a *factor tree* for both 20 and 30 to obtain their *prime factorization*. Draw a vertical separator line between the two factor trees to emphasize they are unrelated.



Create factor trees for **20** and **30**.

<u>STEP 2</u>: List prime factorization for 20 and 30 in a horizontal row. Write the factor lists in order, from smallest to largest. Under the bottom row, draw a line and write "LCM: ".

20: 2 • 2 • 5
<u>30: 2 • 3 • 5</u>
LCM:

Write prime factors in order, smallest to largest.

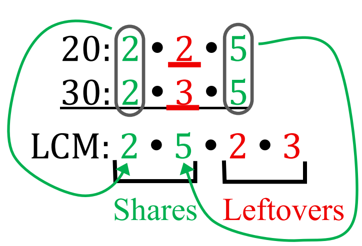
<u>STEP 3</u>: Use the *Shares & Leftovers* technique to determine which factors to include as part of the LCM.

Shares:

- Draw an oval shape around the shared factors (2s and 5s) between the two rows.
  - Shared factors are not required to be directly over each other.
  - Shared factors could be listed offset, resulting in a slanted oval shape.
- Since 2 is shared within the oval shape, we bring down only *one* of the 2s.
- Since 5 is shared within the oval shape, we bring down only *one* of the 5s.

Leftovers:

- These factors are not enclosed in an oval shape since they are different (2 and 3).
- <u>Underline</u> all leftover factors and bring down *all* of them (2 and 3).



Place **Shares** and **Leftovers** into their own "buckets." Then multiply all factors in both buckets for the LCM.

<u>STEP 4</u>: Multiply the *Shares* and the *Leftovers* together 2 • 5 • 2 • 3 for LCM = 60.

Shares & Leftovers borrowed from **Professor Vernamonti**

Add Fractions

Example: Add and simplify $\frac{7}{24} + \frac{15}{36}$

STEP 1 – LCD: To find the **L**owest **C**ommon **D**enominator (**LCD**), we can use either of the two methods for finding the Least Common Multiple (LCM). Examine denominators to determine which method to use. Perhaps 24 and 36 will have a match within the first few multiples. Let's try *List of Multiples* method first. If after the first few multiples we see that 24 and 36 do not have a common multiple, then we can switch to *Shares & Leftovers* method. We write multiples of 24 and 36 and notice that the LCD = 72.

24	36
48	72
72	

List multiples of **24** and **36**.

STEP 2 – Multiply by $\frac{n}{n}$: Multiply numerator *and* denominator ($\frac{n}{n}$) of each fraction that does not originally have the LCD (72) as its denominator. Both denominators are not 72. Thus, we multiply both fractions by some number to make them become the LCD (72). Start with the left fraction. Place a multiplication dot " • " to the left of 24. Ask yourself, "What number times 24 is 72?" Use trial and error to get 3 • 24 = 72. Thus, multiply both denominator *and* numerator by 3. Using the same procedure for the right fraction, we find 2 • 36 = 72. Multiply both denominator *and* numerator by 2.

$$\frac{3 \cdot 7}{3 \cdot 24} + \frac{2 \cdot 15}{2 \cdot 36} \Rightarrow \frac{21}{72} + \frac{30}{72}$$

Multiply numerator and denominator by the same number ($\frac{n}{n}$).

Note: The number we multiply each fraction by will be the same "up and down" because we must maintain the ratio of numerator *to* denominator. However, that number will be different between the two fractions because the original denominators are different.

STEP 3 – Add: We add numerators and keep denominator the same to get $\frac{51}{72}$.

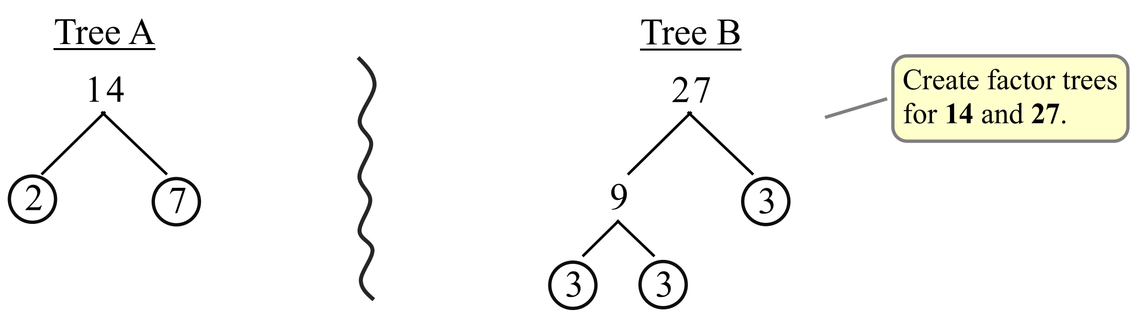
STEP 4 – Reduce: If possible, reduce $\frac{51}{72}$. Both numerator and denominator are big numbers making it difficult to know if this fraction reduces. However, using our knowledge of *divisibility rules,* we find that 51 and 72 are both divisible by 3. Thus, we divide numerator and denominator by 3 to arrive at the answer: $\frac{17}{24}$.

Subtract Fractions

Example: Subtract and simplify $\frac{8}{14} - \frac{9}{27}$

The process of subtracting fractions is identical to adding fractions. Of course, the exception is that in Step 3, we subtract numerators instead of adding them.

STEP 1 – LCD: Examining the denominators, 14 and 27, it seems that we will not get a quick match (within the first few multiples) by using the *List of Multiples* method to find the LCD. We will try the *Shares & Leftovers* method instead. Complete a *factor tree* for both 14 and 27 to obtain their *prime factorization*.



List the prime factorization for 14 and 27 in a horizontal row from smallest to largest. Use the *Shares & Leftovers* technique to determine which factors to include as part of the LCM. There are no *Shares* but instead, all the factors are *Leftovers*. Therefore, bring down *all* factors from both rows.

14: 2 • 7
27: 3 • 3 • 3
LCM: 2 • 7 • 3 • 3 • 3 = 378

> There are no **Shares** factors. All factors are **Leftovers**. Multiply all Leftover factors for the LCM.

STEP 2 – Multiply by $\frac{n}{n}$: Multiply numerator *and* denominator ($\frac{n}{n}$) of each fraction that does not originally have the LCD (378) as its denominator. Both denominators are not 378. Thus, we multiply both fractions by some number to make them become the LCD (378).

Left Fraction (Next page): Place a multiplication dot " • " to the left of 14. Ask yourself, "What number times 14 is 378?" This is difficult with trial and error. However, we will use the *prime factorization* of the LCM from Step 1 to help us. We have 14 in the original denominator. We are looking for other factors that when multiplied by 14 will result in 378. Notice that we also have the 2 • 7 = 14 in the prime factorization 2 • 7 • 3 • 3 • 3. The remaining factors that result in 378 are 3 • 3 • 3 = 27. Thus, we multiply the 14 by 27 to get 378 in the denominator. But we also multiply the numerator by 27 to maintain the fraction's ratio.

Right Fraction: Place a multiplication dot " • " to the left of 27. Ask yourself, "What number times 27 is 378?" Again, we will use the *prime factorization* of the LCM from Step 1 to help us. We have 27 in the original denominator. We are looking for other factors that when multiplied by 27 will result in 378. Notice that we also have the 3 • 3 • 3 = 27 in the prime factorization 2 • 7 • 3 • 3 • 3. Observe that the remaining factors resulting in 378 are 2 • 7 = 14. Thus, we multiply the 27 by 14 to get 378 in the denominator. And we also multiply the numerator by 14 to maintain the fraction's ratio.

$$\frac{27 \bullet 8}{27 \bullet 14} - \frac{14 \bullet 9}{14 \bullet 27} \Rightarrow \frac{216}{378} - \frac{126}{378}$$

Multiply numerator and denominator by the same number ($\frac{n}{n}$).

STEP 3 – Subtract: We subtract numerators and keep denominator the same to get $\frac{90}{378}$.

STEP 4 – Reduce: If possible, reduce $\frac{90}{378}$. Both numerator and denominator are big numbers making reducing difficult. But using our knowledge of *divisibility rules,* we see that 90 and 378 are *even* numbers so they are divisible by 2. Use long division to reduce.

$$\frac{90 \div 2}{378 \div 2} \Rightarrow \frac{45}{189}$$

Divide by the common factor **2** to reduce.

Is $\frac{45}{189}$ the final answer? Use *divisibility rules* again to see. Are 45 and 189 divisible by 2 again? No since both are not even numbers. Are 45 and 189 divisible by 5? No because the ones place value digit of 45 and 189 are not both 0 or 5. Are 45 and 189 divisible by 10? No because the ones place value digit of 45 and 189 are not both 0. We check 3 or 9. For 3, add digits of numerator (4 + 5 = 9) and denominator (1 + 8 + 9 = 18). We see that the sum of the digits of numerator (9) and denominator (18) is divisible by 3. That means that the original numerator (45) and denominator (189) are also divisible by 3. Thus, we can divide numerator and denominator by 3 to reduce.

But let's also check divisibility for 9. Recall that 9 has the same divisibility rule as 3. Observe that 9 can be used to reduce as well. We have a choice to divide either by 3 or 9. We choose to divide by 9 because it will result in a greater reduction.

$$\frac{45 \div 9}{189 \div 9} \Rightarrow \frac{5}{21}$$

Divide by the common factor **9** to reduce.

Is $\frac{5}{21}$ the final answer? We use divisibility rules once again to check. We notice that the fraction does not reduce further. Additionally, our knowledge of the multiplication facts confirms there is no common factor (except 1) between 5 and 21. We are done reducing.

Cautions

Use **caution** with these.

- The **K**eep **C**hange **F**lip (**KCF**) technique is used only for dividing fractions. Do not perform KCF when multiplying, adding, or subtracting fractions. The purpose of KCF is to convert division into multiplication. Then we solve the problem as if it was originally multiplication.
- The **L**owest **C**ommon **D**enominator (**LCD**) is used only for adding or subtracting fractions. Do not try to find the LCD when multiplying or dividing fractions.
- When adding or subtracting fractions, *do not* simplify at the beginning because cross-cancelling is not permitted then. Instead, wait until STEP 4 to reduce.

Your Turn

It is time to demonstrate your mastery of fractions. Write your detailed work then check your answers.

Try these problems.

Convert Mixed Number to Improper Fraction

a) $12\frac{2}{7}$

Convert Improper Fraction to Mixed Number

b) $\frac{43}{6}$

Find All the Factors of a Number

c) Find all the factors of 375.

Find Prime Factorization

d) Find the prime factorization of 350.

Simplify a Fraction

e) Simplify the fraction $\frac{30}{42}$.

Multiply Fractions

f) $\frac{21}{10} \bullet \frac{4}{9}$ g) $64 \bullet \frac{5}{8}$

Divide Fractions

h) $\frac{36}{5} \div \frac{6}{7}$ i) $24 \div \frac{3}{8}$

Find Least Common Multiple (LCM)

j) 25 and 35

Add Fractions

k) $\frac{3}{15} + \frac{10}{25}$

Subtract Fractions

l) $\frac{15}{27} - \frac{5}{45}$

Do not look at the answer below until after you complete the problem.

Answers:

a) $\frac{86}{7}$ b) $7\frac{1}{6}$ c) 1, 3, 5, 15, 25, 75, 125, 375 d) 2 • 5 • 5 • 7

e) $\frac{5}{7}$ f) $\frac{14}{15}$ g) 40 h) $\frac{42}{5}$ i) 64

j) 175 k) $\frac{3}{5}$ l) $\frac{4}{9}$

How did you do?

Tip: Rewrite steps of each problem into your notebook but using your **own words.**

Worksheet

Use the worksheet below to write out your answers.