1

Introduction to Logic Chapter 3

MAT 120

3.1 Statements & Quantifiers

Negations of Quantified Statements

Statement		Negation
All do	$\longleftrightarrow$	Some do not
Some do	$\longleftrightarrow$	None do

3.2 Truth Tables & Equivalent Statements

Truth Table for Conjunction

p and q

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table for Disjunction

p or q

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Tables

Standard Format of Truth Values for 2 Statements

p	q	Compound Statement
T	T	
T	F	
F	T	
F	F	

Standard Format of Truth Values for 3 Statements

p	q	r	Compound Statement
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

De Morgan's Laws for Logical Statements

$\sim(p \vee q) \equiv \sim p \wedge \sim q$ and

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

3.3 The Conditional & Circuits

Truth Table for Conditional

If p, then q

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Negation of $p \to q$

$$\sim(p \to q) \equiv p \wedge \sim q$$

Writing Conditional as Disjunction

$$p \to q \equiv \sim p \vee q$$

3.4 The Conditional and Related Statements

Related Conditional Statements

Conditional Statement	$p \to q$	Equivalent
Converse	$q \to p$	Equivalent
Inverse	$\sim p \to \sim q$	
Contrapositive	$\sim q \to \sim p$	

Common Translations of $p \to q$

If p, then q	p is sufficent for q
If p, q	q is necessary for p
p implies q	All p are q
p only if q	q if p

Biconditionals

$$p \leftrightarrow q \equiv (q \to p) \wedge (p \to q)$$

Truth Table for Biconditional

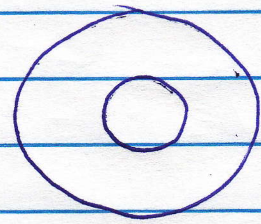
p if and only if q

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

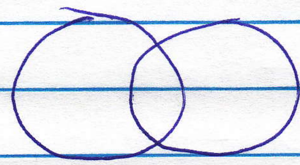
5

3.5 Analyzing Arguments with Euler Diagrams

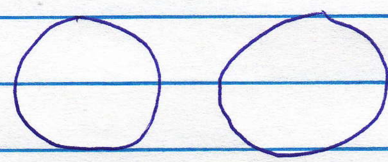
Case 1 "All" ....



Case 2 "Some" ....



Case 3 "No" or "None" ....



3.6 Analyzing Arguments with Truth Tables

Step 1 Assign letter to represent each component statement.

Step 2 Express each premise & conclusion symbolically.

Ex:

Premise 1: $p \to q$

Premise 2: p

Conclusion: q

Step 3 Form symbolic statement of entire argument. Write conjunction ($\wedge$) of all premises as antecedent of a conditional statement and conclusion of argument as consequent.

Ex:

$$[(\underset{\text{premise}}{p \to q}) \underset{\text{and}}{\wedge} \underset{\text{premise}}{p}] \underset{\text{implies}}{\to} \underset{\text{conclusion}}{q}$$

Step 4 Complete truth table. If it is a tautology, argument is valid. Otherwise invalid.

Ex:

p	q	$p \to q$	$(p \to q) \wedge p$	$[(p \to q) \wedge p] \to q$

7

Valid Argument Forms

Modus Ponens	Modus Tollens	Disjunctive Syllogism	Reasoning by Transitivity
$p \to q$	$p \to q$	$p \vee q$	$p \to q$
p	$\sim q$	$\sim p$	$q \to r$
q	$\sim p$	q	$p \to r$

Invalid Argument Forms (Fallacies)

Fallacy of the Converse	Fallacy of the Inverse
$p \to q$	$p \to q$
q	$\sim p$
p	$\sim q$