# *Trigonometric Properties & Theorems 4x6 Card*

**Last Updated: 5/1/06**

| 30° | 45° | 60° | 90° | 120 | 135 | 150 | 180 | | | sin Θ | cos Θ | tan Θ | csc Θ | sec Θ | cot Θ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π | $\frac{\pi}{6}$ | 30 | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | 2 | $\frac{2\sqrt{3}}{3}$ | $\sqrt{3}$ |
| 210 | 225 | 240 | 270 | 300 | 315 | 330° | 360 | $\frac{\pi}{4}$ | 45 | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 | $\sqrt{2}$ | $\sqrt{2}$ | 1 |
| $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{4}$ | $\frac{11\pi}{6}$ | 2π | $\frac{\pi}{3}$ | 60 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ | $\frac{2\sqrt{3}}{3}$ | 2 | $\frac{\sqrt{3}}{3}$ |

$i^1 = i = \sqrt{-1}$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

$y = \log x$ if $x = 10^y$

| | | sin Θ | cos Θ | tan Θ | csc Θ | sec Θ | cot Θ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | ND | 1 | ND |
| $\frac{\pi}{2}$ | 90 | 1 | 0 | ND | 1 | ND | 0 |
| π | 180 | 0 | -1 | 0 | ND | -1 | ND |
| $\frac{3\pi}{2}$ | 270 | -1 | 0 | ND | -1 | ND | 0 |

Period = $T = \frac{2\pi}{\omega}$ Phase Shift = $\frac{\Phi}{\omega}$

Max $\{1, |a_0|+|a_1|+\ldots+|a_{n-1}|\}$, or
$1 + \text{Max}\{|a_0|, |a_1|, \ldots, |a_{n-1}|\}$

$\sin^2\Theta + \cos^2\Theta = 1$ $\tan^2\Theta + 1 = \sec^2\Theta$ $\cot^2\Theta + 1 = \csc^2\Theta$

$y = \log_e x = \ln x$ if $x = e^y$

$\sin\Theta = \frac{b}{r}$ $\cos\Theta = \frac{a}{r}$ $\tan\Theta = \frac{b}{a}$
$\csc\Theta = \frac{r}{b}$ $\sec\Theta = \frac{r}{a}$ $\cot\Theta = \frac{a}{b}$

If $a^u = a^v$, then $u = v$

$\log_a 1 = 0$ $\log_a a = 1$

$\log_a(MN) = \log_a M + \log_a N$

$\sin t = b$ $\cos t = a$ $\tan t = \frac{b}{a}$ $\csc t = \frac{1}{b}$ $\sec t = \frac{1}{a}$ $\cot t = \frac{a}{b}$

$a^{\log_a M} = M$

$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

If $M = N$, then $\log_a M = \log_a N$
If $\log_a M = \log_a N$, then $M = N$

Exponential Function

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

$\log_a M^r = r \log_a M$

$\log_a M = \frac{\log M}{\log a}$ and $\log_a M = \frac{\ln M}{\ln a}$

$y = \log_a x$ if $x = a^y$

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

$\log_a a^r = r$

$y = A\sin(\omega x - \Phi) = y = A\sin[\omega(x - \frac{\Phi}{\omega})]$ watch sign!

Graph begins at *Phase Shift*: $\omega x - \Phi = 0$ or $x = \frac{\Phi}{\omega}$

Graph ends at *Period* + *Phase Shift*: $\omega x - \Phi = 2\pi$ or $x = \frac{2\pi}{\omega} + \frac{\Phi}{\omega}$

$\sin(2\Theta) = 2\sin\Theta\cos\Theta$ $\cos(2\Theta) = \cos^2\Theta - \sin^2\Theta$
$\cos(2\Theta) = 1 - 2\sin^2\Theta$ $\cos(2\Theta) = 2\cos^2\Theta - 1$

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$
$b^2 = a^2 + c^2 - 2ac\cos\beta$
$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$y = \sin^{-1} x$ means $x = \sin y$ where $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$y = \cos^{-1} x$ means $x = \cos y$ where $-1 \le x \le 1$ and $0 \le y \le \pi$

$y = \tan^{-1} x$ means $x = \tan y$ where $-\infty < x < \infty$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Difference of Two Squares: $x^2 - a^2 = (x - a)(x + a)$
Perfect Squares: $x^2 + 2ax + a^2 = (x + a)^2$
$x^2 - 2ax + a^2 = (x - a)^2$
Sum of 2 Cubes: $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$
Difference of Two Cubes: $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Law of Sines*
**Case 1**: ASA or SAA
1-Find 3rd angle, 2-Use 2 ratios to find other 2 sides (cross-multiply).
**Case 2**: SSA (ambiguous) – angle opposite one of the sides known.
1-Find 2nd angle and see if there are 2 solutions (cross-multiply), 2-Find two 3rd angles (if 2 solutions), 3-Find two 3rd sides (if 2 solutions) using ratios. Domain: [-1, 1]

*Law of Cosines*
**Case 3**: SAS – two sides and included angle known.
1-Find 3rd side with formula, 2-Find 2nd angle with formula, 2-Find 3rd angle.
**Case 4**: SSS
1-Find angles $\alpha$ and $\beta$ with formulas, 2-Find 3rd angle.

*Rational Functions: Vertical Asymptotes*
1-Reduce to lowest terms, 2-Cancel, 3-Get vertical asymptotes (check domain restr.)

*Rational Functions: Analyzing Graphs*
1-Cancel factors in num/denom, 2-Find x-int in num, 3-Find y-int by R(0)

*Find Real/Complex Zeros of Function*
**Step 1** – list degree, **Step 2** – Descartes' Rules of Signs, **Step 3** – part a) rational zeros (p/q); part b) synthetic division to find factors, **Step 4** – write in factored form & list zeros.

Courtesy of **George Hartas**
Resource: Algebra & Trigonometry, 7th Edition, Michael Sullivan, 2005, Pearson Education