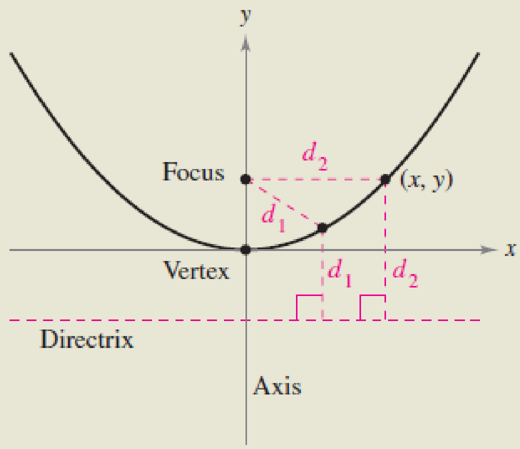
Conic Sections

Parabolas

Definition of a Parabola

A **parabola** is the set of all points (x, y) in a plane that are equidistant from a fixed line called the **directrix** and a fixed point called the **focus** (not on the line). The midpoint between the focus and the directrix is called the **vertex**, and the line passing through the focus and the vertex is called the **axis** of the parabola.



Standard Equation of a Parabola (Vertex at Origin)

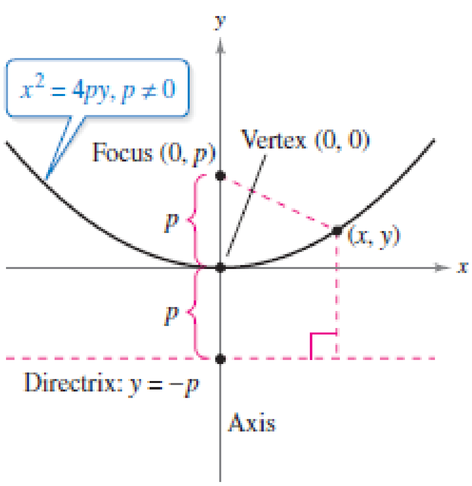
The **standard form of the equation of a parabola** with vertex at $(0, 0)$ and directrix $y = -p$ is given by

$$x^2 = 4py, \quad p \neq 0. \qquad \text{Vertical axis}$$

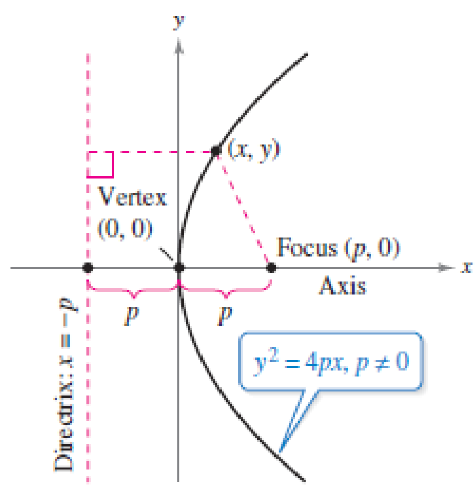
For directrix $x = -p$, the equation is given by

$$y^2 = 4px, \quad p \neq 0. \qquad \text{Horizontal axis}$$

The focus is on the axis p units (directed distance) from the vertex. See Figure B.3.



Parabola with Vertical Axis

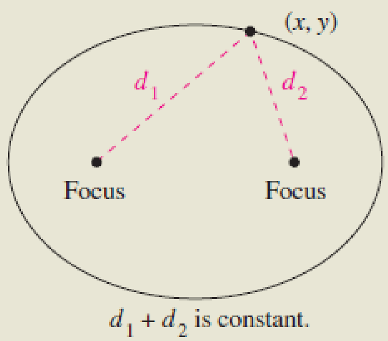


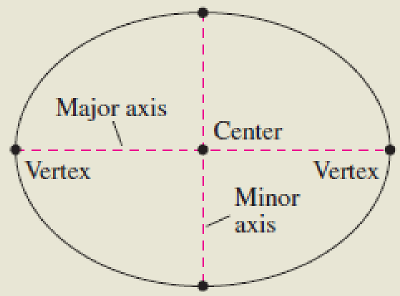
Parabola with Horizontal Axis

Ellipses

Definition of an Ellipse

An **ellipse** is the set of all points (x, y) in a plane the sum of whose distances from two distinct fixed points, called **foci,** is constant.

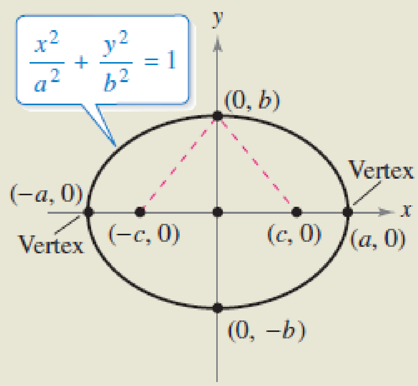




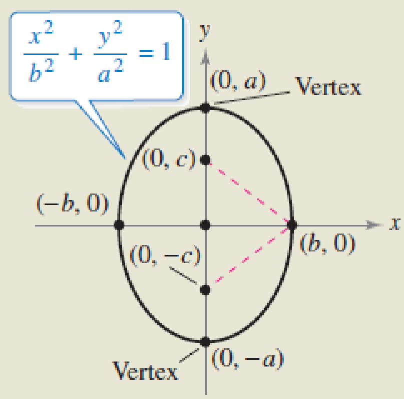
Standard Equation of an Ellipse (Center at Origin)

The **standard form of the equation of an ellipse** with the center at the origin and major and minor axes of lengths $2a$ and $2b$, respectively (where $0 < b < a$), is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$



Major axis is horizontal.
Minor axis is vertical.



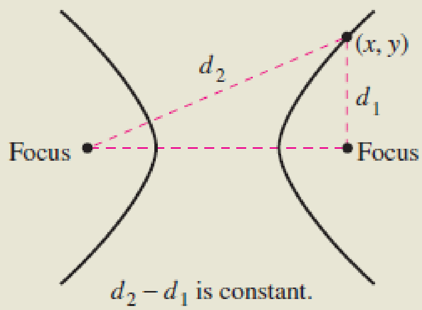
Major axis is vertical.
Minor axis is horizontal.

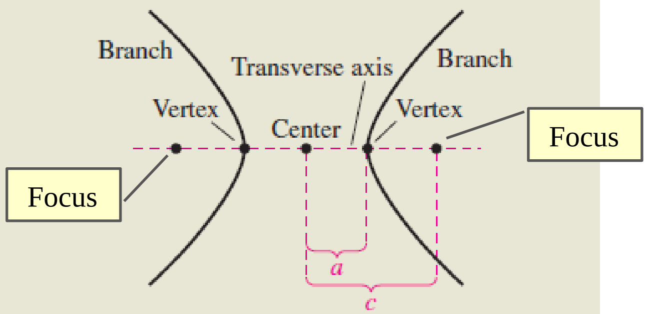
The vertices and foci lie on the major axis, a and c units, respectively, from the center. Moreover, a, b, and c are related by the equation $c^2 = a^2 - b^2$.

Hyperbolas

Definition of a Hyperbola

A **hyperbola** is the set of all points (x, y) in a plane the difference of whose distances from two distinct fixed points, called **foci,** is constant.

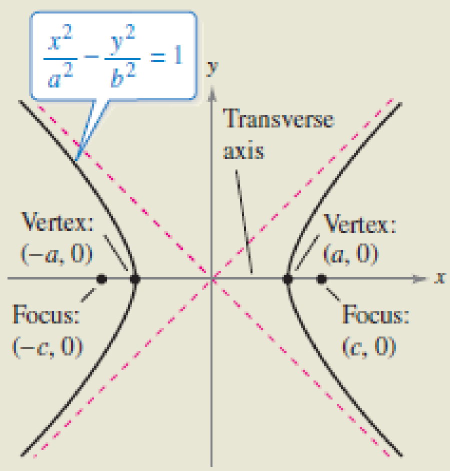


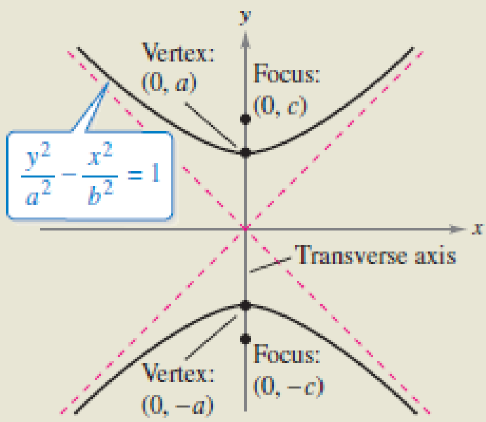


Standard Equation of a Hyperbola (Center at Origin)

The **standard form of the equation of a hyperbola** with the center at the origin (where $a \neq 0$ and $b \neq 0$) is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$$





The vertices and foci are, respectively, a and c units from the center. Moreover, a, b, and c are related by the equation

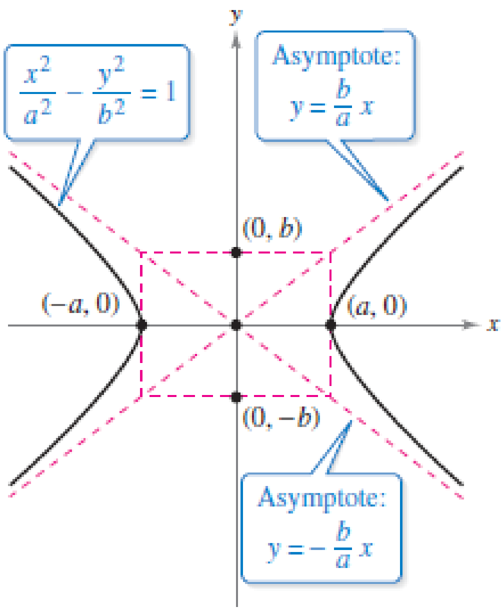
$$\boldsymbol{c^2 = a^2 + b^2}$$

Asymptotes of a Hyperbola

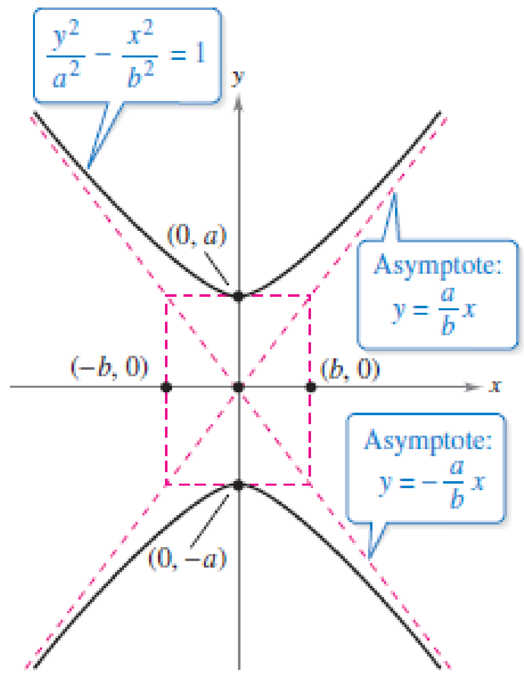
Asymptotes of a Hyperbola (Center at Origin)

$y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$ Transverse axis is horizontal.

$y = \frac{a}{b}x$ and $y = -\frac{a}{b}x$ Transverse axis is vertical.



Transverse axis is horizontal.



Transverse axis is vertical.

Source: www.cengage.com/resource_uploads/downloads/1133105068_339608.pdf