

Definitions of the Hyperbolic Functions:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}, \quad x \neq 0$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{coth} x = \frac{1}{\tanh x}, \quad x \neq 0$$

Hyperbolic Identities:

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh^2 x + \operatorname{sech}^2 x = 1$$

$$\operatorname{coth}^2 x - \operatorname{csch}^2 x = 1$$

$$\sinh^2 x = \frac{-1 + \cosh 2x}{2}$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$$

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$$

$$\cosh^2 x = \frac{1 + \cosh 2x}{2}$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

Inverse Hyperbolic Functions:

Function

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$$

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$$

$$\operatorname{coth}^{-1} x = \frac{1}{2} \ln \frac{x+1}{x-1}$$

$$\operatorname{sech}^{-1} x = \ln \frac{1 + \sqrt{1 - x^2}}{x}$$

$$\operatorname{csch}^{-1} x = \ln \left(\frac{1}{x} + \frac{\sqrt{1 + x^2}}{|x|} \right)$$

Domain

$$(-\infty, \infty)$$

$$[1, \infty)$$

$$(-1, 1)$$

$$(-\infty, -1) \cup (1, \infty)$$

$$(0, 1]$$

$$(-\infty, 0) \cup (0, \infty)$$

Integration Involving Inverse Hyperbolic Functions:

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln(u + \sqrt{u^2 \pm a^2}) + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$$

$$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a} \ln \frac{a + \sqrt{a^2 \pm u^2}}{|u|} + C$$