Vector-Valued Function Formulas

1. The unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{||\mathbf{r}'(t)||} = \dfrac{\mathbf{v}(t)}{||\mathbf{v}(t)||}, \quad \mathbf{r}'(t) \neq \mathbf{0}$

2. The principle unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||}$

3. The tangential component of the acceleration: $a_{\mathbf{T}} = \mathbf{a} \cdot \mathbf{T} = \dfrac{\mathbf{v} \cdot \mathbf{a}}{||\mathbf{v}||} = \dfrac{\mathbf{r}' \cdot \mathbf{r}''}{||\mathbf{r}'||} = \dfrac{d^2s}{dt^2}$

4. The normal component of the acceleration: $a_{\mathbf{N}} = \mathbf{a} \cdot \mathbf{N} = \dfrac{||\mathbf{v} \times \mathbf{a}||}{||\mathbf{v}||} = \dfrac{||\mathbf{r}' \times \mathbf{r}''||}{||\mathbf{r}'||} = K\left(\dfrac{ds}{dt}\right)^2$

   $$\text{or} \quad a_{\mathbf{N}} = \sqrt{||\mathbf{a}||^2 - a_{\mathbf{T}}^2}$$

5. Acceleration: $\mathbf{a} = a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$

6. Arc length: $\displaystyle\int_a^b ||\mathbf{r}'(t)||dt$

7. Curvature in the Plane: $K = \dfrac{|y''|}{[1 + (y')^2]^{3/2}}, \quad C$ given by $y = f(x)$

   $$K = \frac{|x'y'' - y'x''|}{[(x')^2 + (y')^2]^{3/2}}, \quad C \text{ given by } x = x(t),\ y = y(t)$$

8. Curvature in Space: $K = ||\mathbf{T}'(s)|| = \dfrac{||\mathbf{T}'(t)||}{||\mathbf{r}'(t)||} = \dfrac{||\mathbf{v} \times \mathbf{a}||}{||\mathbf{v}||^3} = \dfrac{||\mathbf{r}'(t) \times \mathbf{r}''(t)||}{||\mathbf{r}'(t)||^3}$

9. The initial velocity vector: $\mathbf{v}(0) = \mathbf{v}_0 = ||\mathbf{v}_0|| \cos\theta\mathbf{i} + ||\mathbf{v}_0|| \sin\theta\mathbf{j}$ where $||\mathbf{v}_0||$ is the given initial speed and θ is the given angle.