# *Trigonometric Integrals*

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## Guidelines For Evaluating Integrals Involving Powers of Sine and Cosine

1. When the power of the sine is odd and positive, save one sine factor and convert the remaining factors to cosines. Then expand and integrate. Because u = cos($x$) du = –sin($x$) dx

$$\int \sin^{\overbrace{2k+1}^{\text{Odd}}} x \cos^n x \, dx = \int \overbrace{(\sin^2 x)^k}^{\text{Convert to cosines}} \cos^n x \overbrace{\sin x \, dx}^{\text{Save for } du} = \int (1 - \cos^2 x)^k \cos^n x \sin x \, dx$$

2. When the power of the cosine is odd and positive, save one cosine factor and convert the remaining factors to sines. Then expand and integrate. Because u = sin($x$) du =cos($x$) dx

$$\int \sin^m x \cos^{\overbrace{2k+1}^{\text{Odd}}} x \, dx = \int (\sin^m x) \overbrace{(\cos^2 x)^k}^{\text{Convert to sines}} \overbrace{\cos x \, dx}^{\text{Save for } du} = \int (\sin^m x)(1 - \sin^2 x)^k \cos x \, dx$$

3. When the powers of both the sine and cosine are even and nonnegative, make repeated use of the formulas

Power-Reducing Formulas:

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

and

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

to convert the integrand to odd powers of the cosine. Then proceed as in the second guideline.

## Guidelines for Evaluating Integrals Involving Powers of Secant and Tangent

1. When the power of the secant is even and positive, save a secant-squared factor and convert the remaining factors to tangents. Then expand and integrate.

Because u = tan(*x*)
du = $\sec^2(x)$ dx

$$\int \sec^{\overbrace{2k}^{\text{Even}}} x \tan^n x \, dx = \int \overbrace{(\sec^2 x)^{k-1}}^{\text{Convert to tangents}} \tan^n x \overbrace{\sec^2 x \, dx}^{\text{Save for } du} = \int (1 + \tan^2 x)^{k-1} \tan^n x \sec^2 x \, dx$$

2. When the power of the tangent is odd and positive, save a secant-tangent factor and convert the remaining factors to secants. Then expand and integrate.

Because u = sec(*x*)
du = sec(*x*)tan(*x*) dx

$$\int \sec^m x \tan^{\overbrace{2k+1}^{\text{Odd}}} x \, dx = \int (\sec^{m-1} x)\overbrace{(\tan^2 x)^k}^{\text{Convert to secants}} \overbrace{\sec x \tan x \, dx}^{\text{Save for } du} = \int (\sec^{m-1} x)(\sec^2 x - 1)^k \sec x \tan x \, dx$$

3. When there are no secant factors and the power of the tangent is even and positive, convert a tangent-squared factor to a secant-squared factor, then expand and repeat if necessary.

Because for integral
$\int \tan^2(x)\sec^2(x)$ dx
set u = tan(*x*)
du = $\sec^2(x)$ dx
then
$\frac{\tan^3(x)}{3}$

$$\int \tan^n x \, dx = \int (\tan^{n-2} x) \overbrace{(\tan^2 x)}^{\text{Convert to secants}} \, dx = \int (\tan^{n-2} x)(\sec^2 x - 1) \, dx$$

Because integral of $\sec^2(x)$ is tan(*x*) for the other integral

4. When the integral is of the form

$$\int \sec^m x \, dx$$

*m* = **secant**

where $m$ is odd and positive, use integration by parts, as illustrated in Example 5 in Section 8.2.

5. When the first four guidelines do not apply, try converting to sines and cosines.